Quantum devices

A «quantum device» could be defined as a device whose functionality or principle of operation essentially depends on quantum mechanical effects. One example of such a device is a circuit in which tunnelling of single electrons through the capacitor plays a major role.

Tunneling

Consider an electron with the kinetic energy K approaching a potential energy barrier with a height $U_0 > K$ and a width a. What is the probability of finding the electron on the other side of barrier? If its motion is described by classical mechanics it's zero. However, in the quantum treatment it's a different story.

The Shrödinger equation outside the barrier is

$$-\frac{\hbar^2}{2m}\Psi'' = K\Psi \quad \Rightarrow \quad \Psi'' + k^2\Psi = 0,$$

where $k^2 = 2mK/\hbar^2$. When $x < 0$ the wave function is given by $\Psi_- = e^{ikx} + re^{-ikx}$, where e^{ikx} is indecent de Broglie wave and re^{-ikx} is reflected de Broglie wave. When $x > a$ the wave function is given by $\Psi_+ = te^{ik(x-a)}$, where $te^{ik(x-a)}$ is transmitted electron and $|t|^2$ is the tunneling probability. In the barrier the following Shrödinger equation holds:

$$-\frac{\hbar^2}{2m}\Psi'' + U_0\Psi = K\Psi \quad \Rightarrow \quad \Psi'' - \varkappa^2\Psi = 0,$$

where $\varkappa^2 = 2m(U_0 - K)/\hbar^2$ The coefficient before Ψ changed sign, so we have real exponents instead of complex exponents:

$$\Psi_{\text{in}} = Ae^{\varkappa x} + Be^{-\varkappa x}.$$

To find t, a little algebra is necessary. First, the values of different wave function and its derivative at $x = 0$ should be consistent:

$$\Psi_-(0) = 1 + r, \quad \Psi_{\text{in}}(0) = A + B \tag{1}$$

$$\Psi'_-(0) = ik(1 - r), \quad \Psi'_{\text{in}}(0) = \varkappa(A - B). \tag{2}$$

The same condition for the $x = a$ is required:

$$\Psi_+(a) = t, \quad \Psi_{\text{in}}(a) = Ae^{\varkappa a} + Be^{-\varkappa a} \tag{3}$$

$$\Psi'_+(a) = ikt, \quad \Psi'_{\text{in}}(a) = \varkappa(Ae^{\varkappa a} - Be^{-\varkappa a}) \tag{4}$$

From the first two equations (1,2) we can express that

$$2 = A + B + \frac{\varkappa}{ik}(A - B),$$

and from the last two (3,4) that

$$0 = Ae^{\varkappa a}\left(1 - \frac{\varkappa}{ik}\right) + Be^{-\varkappa a}\left(1 + \frac{\varkappa}{ik}\right).$$

The value of A than could be found:

$$A = \frac{2\left(1 + \frac{\varkappa}{ik}\right)}{\left(1 + \frac{\varkappa}{ik}\right)^2 + e^{2\varkappa a}\left(1 - \frac{\varkappa}{ik}\right)^2}.$$

Using the equations (3,4) again we can express t from A and obtain the answer

$$t = \frac{4e^{\varkappa a}}{\left(1 + \frac{\varkappa}{ik}\right)^2 + e^{2\varkappa a}\left(1 - \frac{\varkappa}{ik}\right)^2}$$

Bow, let's analyze the case of a relatively thick barrier, where $\varkappa a \gg 1$:

$$t = \frac{4e^{-\varkappa a}}{\left(1 - \frac{\varkappa}{ik}\right)^2}.$$

This equation illustrates why tunneling is not observed in the macroscopic world: the probability of tunneling exponentially decays with the width of the barrier.

But what is the characteristic length of tunnlling $1/\varkappa$? A real system that can be described by this equations is a thin gap between two metal films also known as a MIM (metal-insulator-metal) junction. For example, the work function of aluminum is ≈ 4.1 eV. It is the difference between the kinetic energy of free electrons in the metal and the height of the potential barrier (outside air) or $U_0 - K$ in our terms. Then, $1/\varkappa = \sqrt{\hbar^2/(2m(U_0 - K))} \approx 0.1$ nm. That is exactly why devices in which tunneling plays a major role should be extremely small.

Shadow deposition

A tunnel junction is usually a thin oxide layer between two conductors (e.g. MIM junction). The standard method for manufacturing these structures is shadow deposition. This method involves sequentially depositing metal (e.g., aluminum) through a pre-made resist template (see Figs. 1a,b).

Two layers of resist, a material sensitive to the electron beam, are applied to the substrate, which is usually silicon. With focusing an electron beam at the desired point, holes (slots) of the designed shape are cut into the upper layer of resist («pattern»). At the same time, a large cavity is formed in the lower layer («support»), which is much more sensitive to the electron beam (see Fig. 1a).

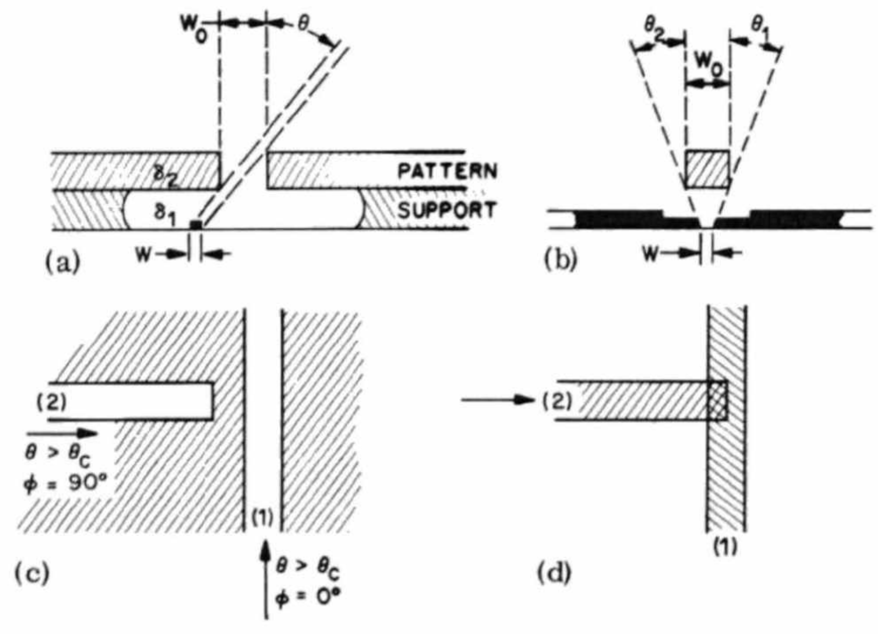


Fig. 1. Shadow deposition. G. J. Dolan, 1977.

Next, the first layer of metal is sprayed (vertical track (1) in Fig. 1c). The deposition is not performed perpendicular to the surface, but at a slight angle.

After that, the surface is oxidized to create a thin layer of insulator. Then deposition is repeated at an angle to the plane to create a horizontal track (2) in Fig. 1c which overlaps the vertical track slightly.

This results in the structure shown in Fig. 1d, consisting of two conductors with a layer of oxide (insulator) between them at the point of contact.

However, this also results in secondary parts of the structure appearing (a «shadow» from the slit (2) in the template during the first spraying, and vice versa). The template designer's task is to ensure that these secondary parts do not affect the structure of the electrical circuit being manufactured.

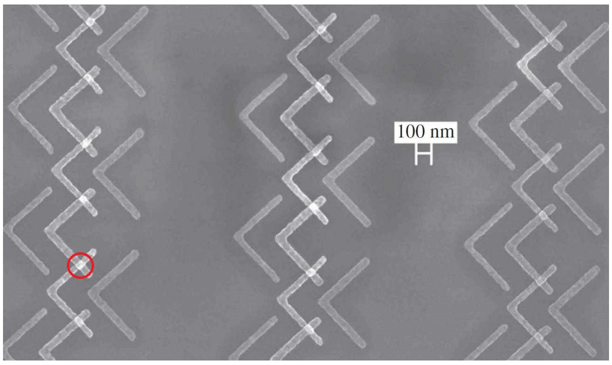


Figure 1. There are several tunnel junctions connected in series. One junction is circled in red. The figure shows three chains of tunnel junctions. The «check marks» that are not connected to each other are a side effect of shadow deposition technology. The picture was taken using scanning electron microscopy. M. Meschke et al., 2015

In the following problems, we will discuss a regulated current source and a thermometer assembled on the basis of tunnel junctions.

T8. Single-electron pump

In this problem, for simplicity, we will consider an electron to be a **positively** charged particle with a charge of $e = 1.6 \cdot 10^{-19}$ C.

Part A. Tunnel junction and Coulomb's blockade

A tunnel junction can be imagined as a parallel connection of a capacitor and a nonlinear resistance. The current through the resistance describes the tunneling of elementary charges from one plate to another if this flow is energetically favourable. In this part of the problem, we will derive the condition for tunneling, that is, we will determine under what conditions the resistance is finite and equal to R_t, and when it becomes infinite. Consider a tunnel junction disconnected from the voltage source.

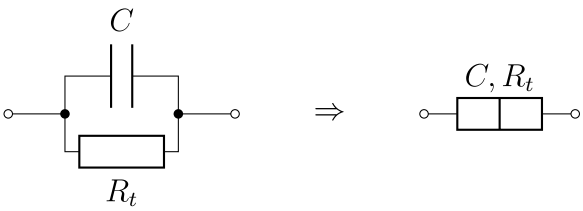


Fig. 1. Schematic diagram of a tunnel junction (left) and its representation in electrical circuit diagrams (right). The nonlinear element corresponds to a constant resistance R_t when tunneling is energetically favourable and to an open circuit (infinite resistance) when tunneling is impossible.

A1	Write down the expression for the energy W of a capacitor with capacitance C (in the equivalent circuit of a tunnel junction) if the plates have a charge of n electron charges. Now let the value n increase or decrease by one. How will the energy of the capacitor change?

Fig. 2. A tunnel junction that is not connected to a source. As a result of tunneling, the charge on the electrodes changes from n to $n+1$.

A2	Assuming that the value of n is not necessarily an integer, determine the values of voltage across the capacitor at which tunneling of single electron is impossible in either the forward or reverse direction.

The existence of a voltage range at which electron tunneling is impossible is called Coulomb blockade.

Part B. Single-electron pump

Using tunnel junctions, it is possible to assemble a so-called single-electron pump, which is capable of generating a direct current.

The figure shows a diagram of such a pump, consisting of three tunnel junctions and two capacitors (« gates »). Two ideal ammeters are included in the circuit to monitor the current flow. There are two points in this circuit where the charge can only change due to tunneling through the junctions. We will call these points « islands » and assume that they can only contain an integer number of electrons: n_1 and n_2, respectively. Depending on the voltages U_1 and U_2 applied to the gates, different numbers of electrons on the islands will be in equilibrium (energy-efficient). Fig. 3b shows the stability regions for each state (we characterize the states by the number of electrons on the islands). The line above the number indicates a minus sign.

Let us assume that tunneling of one electron, i.e., transition between neighbouring states (across any boundary on the graph), requires time τ_0.

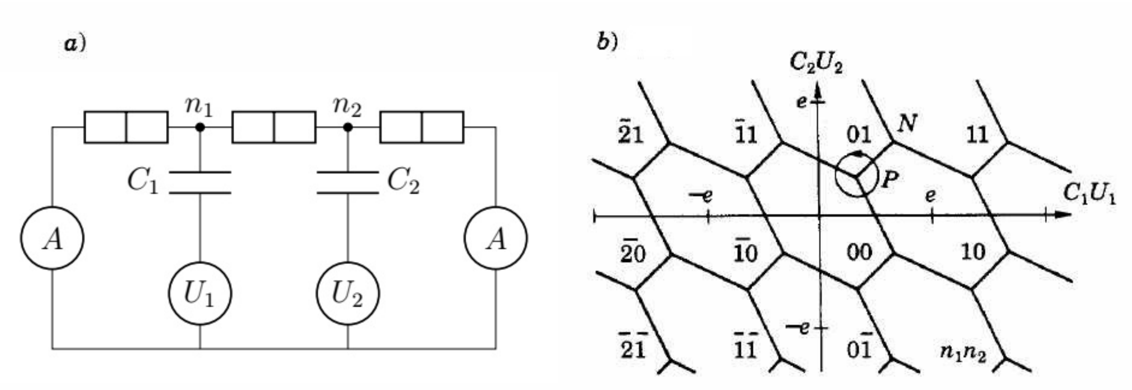


Fig. 3. Single-electron pump and its state diagram

Let the voltages across the gates depend on time according to the equations

$$\begin{cases} U_1(t) = U_1^{\text{d.c.}} + u \exp\left[2\pi i f t\right], \\ U_2(t) = U_2^{\text{d.c.}} + u \exp\left[2\pi i f t + i\varphi\right]. \end{cases}$$

Let us assume that $f \ll \tau_0^{-1}$. Then, after averaging over the period, the current value will be determined precisely by the frequency f of the signal at the gates.

B1	At what value of φ will the system in coordinates U_1 — U_2 move counterclockwise in a circle around point P (see Fig. 3b)? What will be the direction of the current through the left ammeter?

B2	How will the answers to the questions in the previous paragraph change if the center of the circle is moved to point N?

The graph below shows the dependence of the current flowing through the left ammeter on the DC component of the voltage at the first gate, with a constant DC voltage at the second gate and $u = 0.3$ mV.

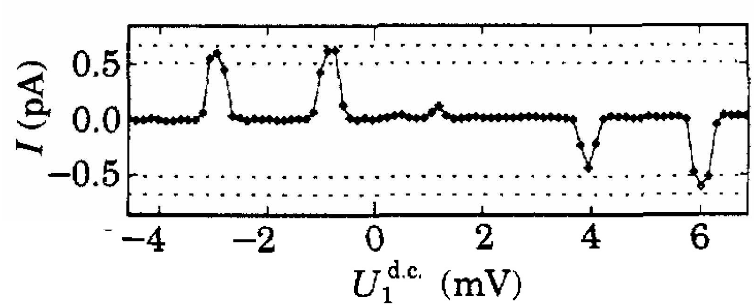


Fig.4. Dependence of the flowing current on the constant component of voltage at the first gate (H. Pothier et al., 1992)

B3	Using the data in the graph, estimate the frequency f of the signal at the gates.

Thus, by setting constant voltage components on the gates and changing the frequency of the variable component, it is possible to regulate the current flowing in the circuit.

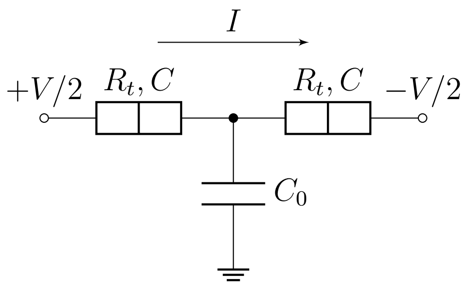
T9. Quantum thermometer

In the case $e^2/C_\Sigma \ll k_\mathrm{B}T$, when the tunnel junction can serve as an element of a primary thermometer. That is, a thermometer that does not need to be calibrated beforehand.

In the simplest case, such a thermometer is a single-electron transistor — two tunnel junctions and a gate (see Fig.1). By recording the conditions and probabilities of tunneling, at sufficiently high temperatures, we can obtain an expression for the current flowing through the tunnel junctions

$$I = \frac{V}{2R_\mathrm{t}} + \frac{e}{R_\mathrm{t}C_\Sigma} \cdot \left[f\left(\frac{eV}{2k_\mathrm{B}T}\right) - f\left(-\frac{eV}{2k_\mathrm{B}T}\right)\right],$$

where $f(x) = \dfrac{1+(x-1)e^x}{(1-e^x)}$. At very high temperatures (e.g., room temperature), the second term can be neglected, and Ohm's law remains valid for two tunnel junctions with resistance R_t connected in series.



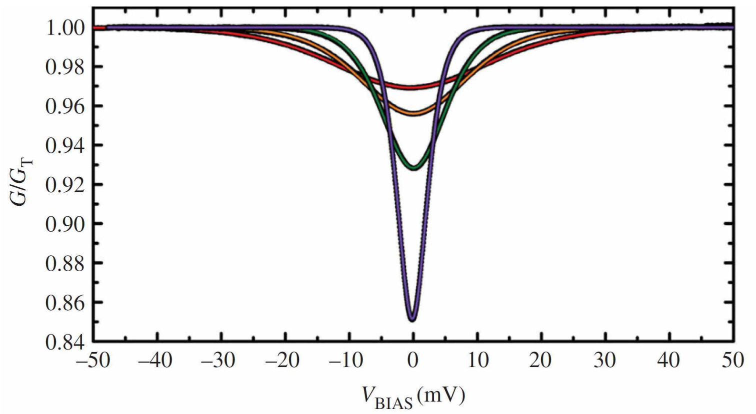
A1	Express the differential conductivity $G = dI/dV$ as $$G/G_\mathrm{t} = 1 - \frac{e^2/C_\Sigma}{k_\mathrm{B}T} \cdot g\left(\frac{eV}{2k_\mathrm{B}T}\right),$$ where $G_\mathrm{t} = (2R_\mathrm{t})^{-1}$ — conductivity of the system at high (room) temperature. Obtain the explicit form of the function $g(x)$ and plot its graph.

A2	Determine the depth of the differential conductivity gap$1 - G_\mathrm{min}/G_\mathrm{t}$ at $V = 0$. Remember that $e^x = \sum x^n/n!$.

A3	Numerically determine the full width $x_{1/2}$ of $g(x)$ on the half-height: $$g\left(\frac{1}{2}x_{1/2}\right) = \frac{1}{2}g(0).$$

A4	Obtain the numerical value of the width of the conductivity gap $V_{1/2}$ at the boiling point of liquid helium $T = 4.21$ K.

Thus, the width of the differential conduction gap depends only on temperature, which allows a single-electron transistor to be used as a primary thermometer.



M. Meschke et al., 2015

A5	The graph shows the dependence of normalized conductivity on half the voltage across the transistor ($V_{\text{BIAS}} = V/2$). For each of the four dependencies, determine the temperature at which it was studied.

A6	Determine the total capacitance C_Σ of a single-electron transistor for which the above dependencies were obtained.