T3. Quantum well with electrons

A classic problem in quantum mechanics is a one-dimensional well with infinite walls. Such a system can be used to describe the behavior of electrons in a very thin metal film.

Consider the following potential:

$$U(x) = \begin{cases} +\infty, & x < 0 \text{ or } x > a \\ 0, & 0 < x < a \end{cases},$$

where a is the width of the well.

For such a system, the probability of finding a particle outside the boundaries of the well is zero, and therefore $\Psi(0) = \Psi(a) = 0$.

A1	Find the solutions Ψ_n of the Schrödinger equation: $$-\frac{\hbar^2}{2m}\Psi'' + U(x)\Psi = E\Psi.$$ Write down corresponding energies E_n.

Let's imagine that there are N electrons in the well that dont interact with each other (for example the elecrtostatic interaction is screened in the metal). Remember that electrons are fermions and that's why they cannot be in the same state.

A2	Calculate the energy E of the whole electrons.

Fermions cannot be in the same state and that's why part of elecrtons «forced» to be on the levels with higher energy. From the classial point of view it's interaction, which calls exchange interaction.

A3	Determine the force F of pressure of electons on the each wall of the well.

T4. Ramsauer–Townsend effect

In 1921, German physicist Karl Ramsauer observed that at certain energies, electrons scattered abnormally little off argon atoms. This effect cannot be explained in terms of classical mechanics, so it played an important role in popularizing quantum mechanics in its early stages of development (Schrödinger's equation was first published five years after the discovery of the Ramsauer effect).

Consider a well of finite depth U_0 and width a:

$$U(x) = \begin{cases} 0, & x < 0 \text{ or } x > a \\ -U_0, & 0 < x < a \end{cases}$$

Let an electron with mass m and momentum $\hbar k$ fall on it, i.e., its wave function is a de Broglie wave:

$$\Psi = e^{-ikx}$$

This wave is partially reflectes from the well and partially transmits trough it. In other words, when $x < 0$:

$$\Psi_- = e^{-ikx} + re^{ikx},$$

where r is the reflection coefficient. When $x > 0$:

$$\Psi_+ = te^{-ikx},$$

where t is the transmission coefficient.

A1	Show that in the well the wavefunction Ψ_{in} is $$\Psi_{\text{in}} = Ae^{iqx} + Be^{-iqx}.$$ Express q in terms of k, m and U_0.

The wave function and its derivative (if there are no infinitely large jumps in potential energy) must be continuous.

A2	Suggest the problem about wave optics, which is completely analogous to the one currently being considered.

A3	Find $\|t\|^2$ — the probability that the particle transmits through the well.

A4	Sketch the graph of $\|t\|^2$ vs k.