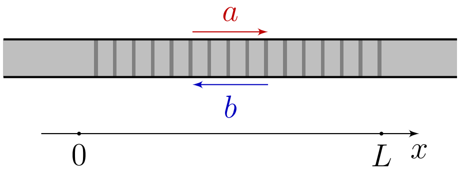
T6. Fiber Bragg grating mirror

Consider a fiber where a Bragg grating of length L is fabricated. According to the theory of coupled modes the amplitudes of the forward wave a and backward wave b are related by the following equations:

$$\begin{cases} \dfrac{da}{dz} = i\kappa b \\ \dfrac{db}{dz} = -i\kappa a \end{cases},$$

where κ is the strength of the grating.



A1	Reduce the system of two differential equations for two amplitudes a and b to a single equation for amplitude a.

Let's consider that the complex amplitude of the wave incident from the left is A_0.

A2	What boundary conditions are there for $a(0)$, $a(L)$, $a'(0)$ and $a'(L)$? *Remark*: The existence of boundary conditions isn't necessary for all given values.

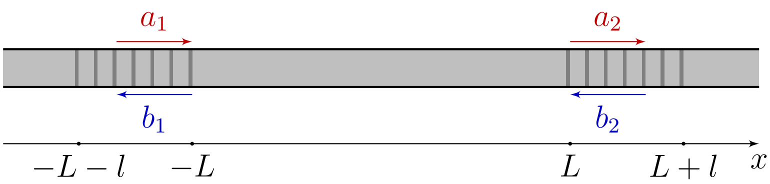
A3	Obtain the value of the reflection coefficient R.

A4	Does the energy loss occur due to the interaction of light with Bragg grating?

T7. Fabry-Perot cavity in fiber

In this problem, we will study the eigenmodes of a resonator based on a fiber Bragg grating.

Let there be two Bragg gratings with length l and grating strength κ manufactured in an optical fiber at a distance of $2L$ from each other.



Within the framework of the theory of coupled modes, we will find solutions $a_{1,2}(x)$, $b_{1,2}(x)$ that do not require an external source: $a_1(-L-l) = 0$, $b_2(L+l) = 0$, and can have any amplitude.

The refractive index of the optical fiber is n.

A1	Write down the relationship between $a_1(-L)$ and $a_2(L)$, $b_1(-L)$ and $b_2(L)$ based on considerations of phase shift when a wave travels along an optical fiber. The answer may contain L, n, and the wave number $k = \omega/c$, where ω is the frequency of light and c is the speed of light.

A2	Express $a_1(-L)$ in terms of $b_1(-L)$. The answer may contain κ, l.

A3	Express $b_2(L)$ in terms of $a_2(L)$. The answer may contain κ, l.

The resulting equations can be reduced to the system

$$\begin{cases} a_2(L) + Ab_1(-L) = 0 \\ Ba_2(L) + b_1(-L) = 0, \end{cases}$$

which has non-trivial solutions (i.e., solutions with any amplitude) when the self-consistency condition $1 - AB = 0$ is satisfied.

A4	Write down the condition for self-consistency when $\kappa l = \infty$. Find the frequencies of the resonator's eigenmodes ω_m, assuming that κ and n do not depend on the wavelength.

The resulting self-consistency condition cannot be satisfied exactly if κl is a finite number. This is because, in this case, the modes has a certain lifetime that can be found. To do this, we take into account that the amplitudes of all waves in the resonator decay as $e^{-\omega'' t}$. For simplicity, we will assume that $L \gg l$.

A5	Modify the equations found in question **A1** to account for wave deacy during the propagation time from one grating to another. From the new self-consistency condition, find the relationship between ω' and κl at $\kappa l \gg 1$ for each eigenmode of the resonator.

A6	Find the quality factors of the resonator's eigenmodes Q_m, assuming that κ and n do not depend on the wavelength.