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T17. Oblique shock wave

A shock wave is a type of gas motion where there exists a certain surface, a front, across which the gas abruptly changes its state. Shock waves occur during explosions or when aircraft travel at supersonic speeds. Similar processes also occur in astrophysics: for example, the Earth's magnetic field, standing in the path of the solar wind, creates a shock wave in it.

Shock waves are divided into three types: normal shock waves, oblique shock waves, and bow shock waves. In a normal shock wave, the gas approaches the boundary perpendicularly to it. In an oblique wave, the gas velocity is directed at an angle to the boundary. A bow shock wave arises when bodies of irregular shape move in a stationary gas at a speed greater than the speed of sound. Such a wave has a shape that is more conical than flat. Describing such a wave is much more complex than the first two, although the formulas obtained when describing an oblique wave can be used when describing local sections of the bow wave.

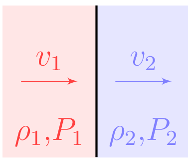
Any shock wave is characterized by Mach number M. if the speed of the wave boundary is v, then the Mach number is defined as $M = v/c$, where c is the speed of sound, which can be calculated as $c = \sqrt{\gamma P/\rho}$. It leads us to the formula $M^2 = v^2 \frac{\rho}{\gamma P}$.

$$\frac{\rho v^2}{P} = \gamma M^2$$

In all parts of the problem, we will be located in the frame of reference of the boundary, which greatly simplifies the calculations.

Part A. Normal shock wave

Lets consider a normal shock wave. The velocity of undisturbed air in this frame of reference is v_1, its pressure and density are P_1 and ρ_1, respectively. After passing through the front, the air parameters undergo a sharp jump and become equal to: v_2, P_2, ρ_2.



A1	Using the mass conservation law, show that $\rho_1 v_1 = \rho_2 v_2$.

A2	By writing down the law of conservation of momentum, get that $P_1 - P_2 = \rho_1 v_1 (v_2 - v_1)$

To the two equations obtained, we can add the law of conservation of energy for a gas flow, also called the Bernoulli equation:

$$\frac{\gamma}{\gamma - 1}\frac{P_1}{\rho_1} + \frac{v_1^2}{2} = \frac{\gamma}{\gamma - 1}\frac{P_2}{\rho_2} + \frac{v_2^2}{2}$$

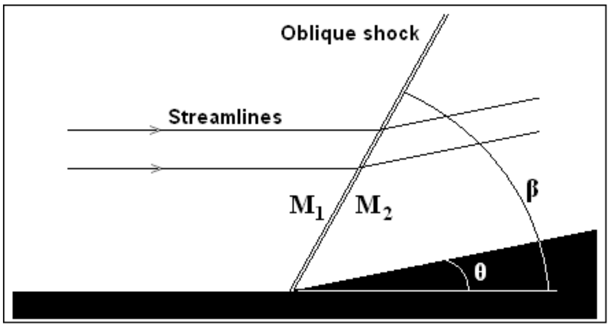
Where γ is the gas adiabatic index. We have three equations and three unknowns - (v_2, P_2, ρ_2). This allows us to calculate each of them. First of all, we will be interested in the velocity of the perturbed gas v_2.

A3	Show that the relation between speeds is as follows: $$k = \frac{v_2}{v_1} = \frac{M_1^2(\gamma - 1) + 2}{M_1^2(\gamma + 1)},$$ where $M_1 = \frac{v_1}{\sqrt{\gamma P_1/\rho_1}}$ - Mach number of the shock wave.

A4	Show that the ratio of pressures on different sides of the front is equal to: $$\frac{P_2}{P_1} = \frac{2\gamma M_1^2 - (\gamma - 1)}{\gamma + 1}$$

Part B. Oblique Shock Wave

Let's study the oblique shock wave that occurs when air strikes an infinite inclined plane with an angle of inclination θ. Consider that the air parameters to the left and right of the border are uniform. (The parameter jump occurs only at the wave boundary) . From this condition, it is clear that the air velocities should be directed parallel to the corresponding surfaces. That's why θ is also called the turn angle - it's the angle between velocities before and after oblique shock.



B1	Prove that the tangential component of the air velocity is preserved when crossing the border.

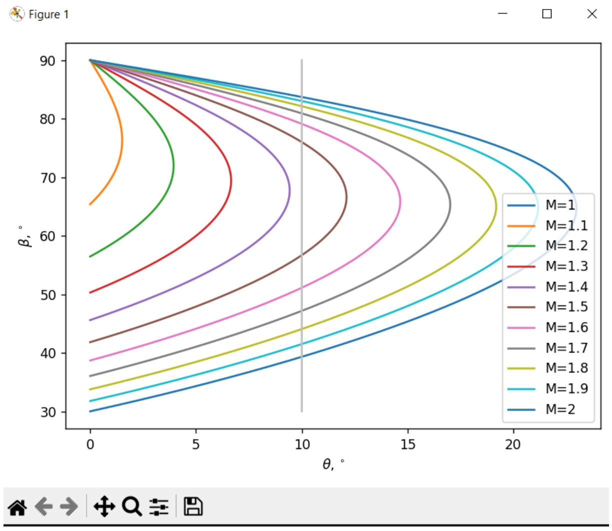
The equations obtained in Part A remain true with one correction: it is the normal velocity components that we use in each of them. This can be seen both from the equations themselves and from the following reasoning: it is possible to find such a frame of reference that the tangential components of air velocities will be equal zero. Then, repeating the calculations from part A in such a frame of reference, we get:

$$k = \frac{v_{n1}}{v_{n2}} = \frac{M_{1n}^2(\gamma - 1) + 2}{M_{1n}^2(\gamma + 1)},$$

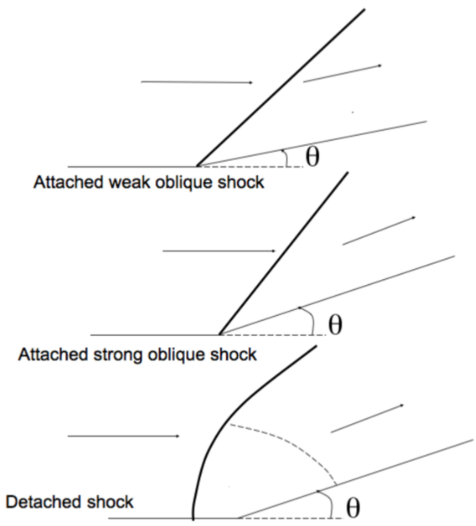
where $M_{1n} = M_1 \cdot \sin\beta$ - is the normal component of the Mach number to the left of the boundary.

B2	Using the above results, get the so-called $\theta - \beta - M$ equation: $$\tan\theta = 2\cot\beta \frac{M_1^2 \sin^2\beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2} \qquad (1)$$

B3	Use the program to plot β graphs from θ at various M_1. For $\theta = 10°$ find M_{min}, at which the solution of equation (1) for the angle β still exists. The values of M are set in the array M_{values}. For convenience, the vertical line $\theta = 10°$ is also drawn on the chart. Program output for the values of $M_{values} = [1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0]$ is shown below.



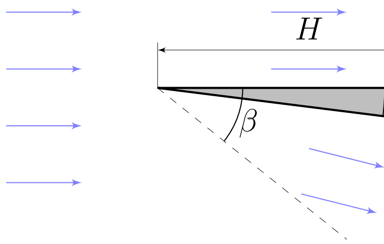
There are no solutions for $M < M_{min}$. This means that the type of shock wave will become different - our model won't work. In fact, the following will happen: since the air is flying at a lower speed, the air that has already collided with the surface will have time to slow down the air flying after it. We get that the shock wave should start before the air reaches the plane. This type of shock wave is called a detached shock. It is depicted in the last picture. If there is a solution for β, then there are two roots. One corresponds to a larger pressure drop, the other to a smaller one. In a real situation, both options can be implemented.



Part C. Supersonic Plane

The $\theta - \beta - M$ equation obtained above can be used to estimate the lift force acting on a supersonic aircraft.

The first aircraft in the world to break the sound barrier was the Bell-X-1. It has wings perpendicular to the body and almost flat. Let's imagine them as an inclined plane with a solution angle of $\theta = 10°$, base $H = 1.5$ m, and length $L = 10$ m (L is the wingspan). The weight of the aircraft is $M = 5000$ kg.



In the reference frame of the aircraft, the flying air is divided into two streams: above the wing and under the wing. Consider that the upper surface of the wing is strictly horizontal and the air above it does not form a shockwave. The air under the wing forms an oblique wave, which we studied in Part B. The air pressure in this wave will be higher than atmospheric pressure, which will create vertical lift.

Let the plane fly at an altitude of 20 kilometers above sea level at a speed of $v_1 = 600$ m/s. Atmospheric pressure at this altitude is $P_1 = 5529$ Pa, air density $\rho_1 = 0.089$ kg/ m^3. The gravitational acceleration is $g = 9.7 \text{m/s}^2$.

C1	What is the mach number of the aircraft M_1?

C2	Graphically find two solutions to the $\theta - \beta - M$ equation - β_1 and β_2. Use a smaller root in next questions.

C3	What is the air pressure under the wing of the aircraft P_2?

C4	Find the vertical force F acting on the wing. With what additional cargo ΔM can the aircraft fly at the given altitude?

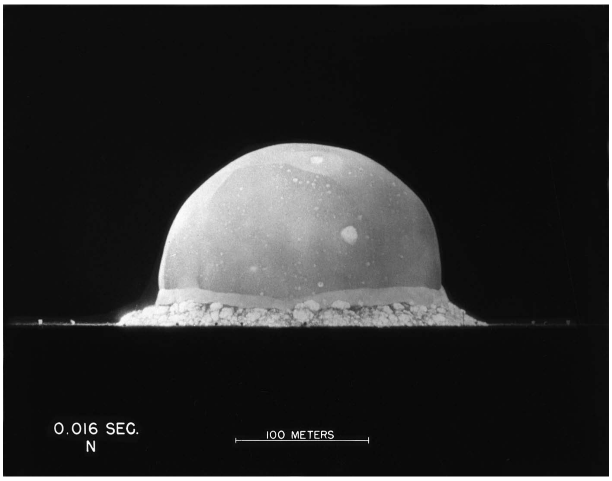
T18. Trinity

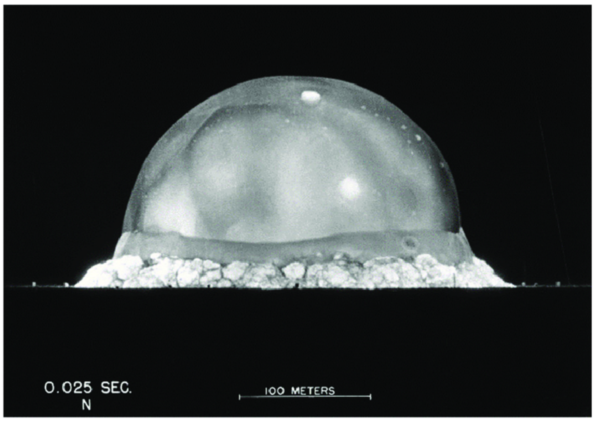
In an environment with a density of ρ_0, a point explosion with an energy of E_0 occurred, which caused an expanding shock wave with a radius of R. In the first approximation, the dependence $R(t)$ will be power-law for each of the arguments ρ_0, E_0, t:

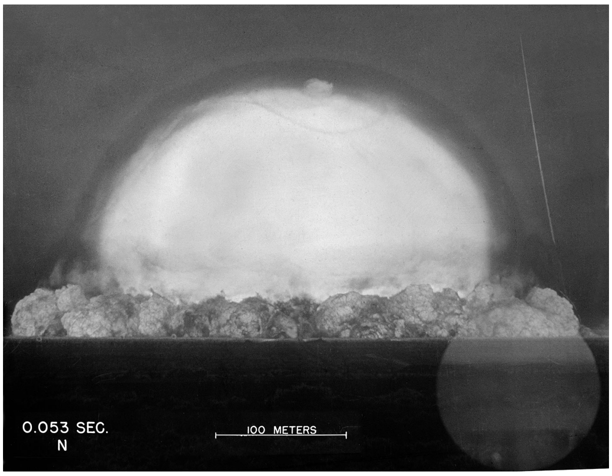
$$R = S(\gamma) \cdot E_0^a \cdot \rho_0^b \cdot t^c,$$

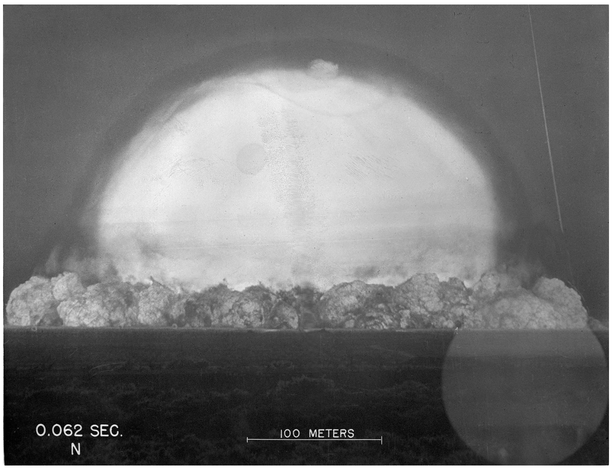
where $S(\gamma)$ is a dimensionless constant depending on the gas.

A1	Analyzing the dimensions, find a, b and c.









A2	Above are photos of the first atomic explosion, taken milliseconds after detonation. At each photo the scale and time from the beginning of the explosion are indicated. Having linearized the dependence from point A1, find the energy E_0 that the atomic bomb released. The air density is $1.3kg/m^3$. The coefficient S for air is $S = 1.03$.